

THE KAVERY ENGINEERING COLLEGE*(An Autonomous Institution)***B.E/B.TECH DEGREE END SEMESTER THEORY EXAMINATIONS, APRIL-MAY 2025***First Semester***MA3151/UMAT24102 - Matrices and Calculus***(Common to all Branches)***Regulations – 2021 & 2024****Duration : 3 Hrs****Maximum : 100 Marks****PART – A (ANSWER ALL QUESTIONS) (Marks 10*2=20)**

Q.No	Questions	BLT	CO	Marks
1.	If the product of two Eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third Eigenvalue of A.	L3	1	2
2.	Find the nature of a quadratic form whose matrix is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.	L3	1	2
3.	Find the derivative of $y = \frac{\cos x}{1 - \sin x}$.	L3	2	2
4.	Find the critical value of the function $f(x) = 6 - 2x - x^2$.	L3	2	2
5.	If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.	L2	3	2
6.	If $x = u^2 - v^2$ and $y = 2uv$, then find $\frac{\partial(x,y)}{\partial(u,v)}$.	L2	3	2
7.	Evaluate $\int \frac{\sin(\log x)}{x} dx$.	L2	4	2
8.	Determine whether the integral $\int_1^\infty \frac{1}{x^2} dx$ is convergent or divergent.	L2	4	2
9.	Sketch the region of integration in $\int_0^1 \int_0^x f(x,y) dx dy$.	L2	5	2
10.	Evaluate $\int_0^1 \int_0^2 dx dy$.	L3	5	2

PART – B (ANSWER ALL QUESTIONS) (Marks 5*16 = 80)

Q.No	Questions	BLT	CO	Marks
11. a	Verify the Cayley – Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ and also find A^{-1} .	L3	1	16
	Or			
b	Find the Canonical form of the QF: $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ through an orthogonal transformation and find the rank, index, signature and nature of the form.	L3	1	16

12.	a	If $f(x) = x^4 - 8x^2 + 16$, Find the local maximum and local minimum.	L2	2	16
		Or			
	b i	Find the absolute maximum and minimum values of the function $f(x) = x^3 - 3x + 1, [0,3]$.	L2	2	8
	i	Show that f is continuous on $(-\infty, \infty)$, $f(x) = \begin{cases} \sin x, & x < \frac{\pi}{4} \\ \cos x, & x \geq \frac{\pi}{4} \end{cases}$	L2	2	8
13.	a i	Find the extreme value of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.	L2	3	8
	i	A rectangular box open at the top, is to have a volume of 32cc. Find the dimensions of the box, that requires the least material for its construction.	L2	3	8
		Or			
	b i	Expand $e^x \sin y$ in powers of x & y up to the third degree terms.	L2	3	10
	i	If $u = x - y, v = y - z, w = z - x$. then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	L2	3	6
14.	a i	Evaluate $\int x^2 \sin x \, dx$ using integration by parts.	L3	4	8
	i	Evaluate $\int \frac{x+2}{(x-1)(x+1)} \, dx$, using by partial fraction method.	L3	4	8
		Or			
	b	Find the Reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x \, dx, n \geq 2$ is an integer.	L3	4	16
15.	a i	Find the area enclosed by the curves $y^2 = 4x$ and $x^2 = 4y$ using double integration.	L3	5	8
	i	Change the order of integration $\int_0^1 \int_0^x xy \, dy \, dx$, and hence evaluate.	L3	5	8
		Or			
	b	Find the Volume of the sphere $x^2 + y^2 + z^2 = a^2$, using triple integration.	L3	5	16